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13. ABSTRACT (Maximum 200 words) A principal goal of modern reliability analysis is to design methods for appraising the reliability of large, logically complex systems that must operate at exceptionally high levels of reliability, but are subject to failure at uncertain times in uncertain ways. Nuclear power plants, the space shuttle, DARPA net and fly-by-wire aircraft are examples of such systems. Current practice is to design a probabilistic model of the stochastic behavior of a complex system and to augment it with numerical appraisal of the most important properties of the joint probability law that the model entails. A very different approach is to model the logical structure of a system so as to identify all logically obtainable events and to not model a probability law that governs event occurrence exactly. In place of assigning a specific probability law to a sample space that contains all obtainable system events, ask system analysts to assign numerical probabilities to some events that lie within their domain of expertise. This recasts the problem and changes the focus of the computational task.				
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Subject: FINAL REPORT

**Prediction and Inference with Incomplete Probabilistic
Knowledge**
AFOSR Grant #F49620-96-1-0307
Final Report

Status of Effort and Background Description

A principal goal of modern reliability analysis is to design methods for appraising the reliability of large, logically complex systems that must operate at exceptionally high levels of reliability, but are subject to failure at uncertain times in uncertain ways. Nuclear power plants, the space shuttle, DARPA net and fly-by-wire aircraft are examples of such systems.

Current practice is to design a probabilistic model of the stochastic behavior of a complex system and to augment it with numerical appraisal of the most important properties of the joint probability law that the model entails. A very different approach is to model the logical structure of a system so as to identify all logically obtainable events and to *not* model a probability law that governs event occurrence exactly. In place of assigning a specific probability law to a sample space that contains all obtainable system events, ask system analysts to assign numerical probabilities to some events that lie within their domain of expertise. This recasts the problem and changes the focus of the computational task.

For a fully specified model, the task is:

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Given initial conditions, a joint probability law for events and either certain or probabilistic knowledge of model parameters, calculate the probability of occurrence of one or more critical events.

The task for the alternative is:

Given the logical structure of obtainable events and some numerical assessments of probabilities of these events, compute bounds on the probabilities of one or more critical events that are not directly assessed *without attempting to specify a joint probability law for all events.*

A sound method for processing complex system probability assessments should:

- ❑ Allow determination of the coherence or incoherence of assessments,
- ❑ Enable computation of coherent bounds on probabilities of events not directly assessed,
- ❑ Allow for efficient revision of bounds in light of additional information in the form of expert judgment or in the form of observation of the occurrence of an event or its complement. It must also allow for efficient revision when additional assessments are provided.
- ❑ Should not contradict Bayesian conditionalization,
- ❑ Be computationally tractable for realistic problems of moderate to large size, and
- ❑ Be based on reasonable assumptions about qualitative features of the probability law governing uncertain system events.

De Finetti's Fundamental Theorem of Probability (FTP) provides both a conceptual and computational framework for carrying out the second of the above programs for calculating probabilities of critical reliability events. In addition, the FTP meets all of the conditions for a sound method just described. It is the foundation for a program of analysis of the reliability of complex systems that exploits and extends recent research at the intersection of probabilistic logic and mathematical programming.

Until recently, the FTP was of primary conceptual value because the FTP prescribes linear and/or non-linear programming problems with exponentially many decision variables, problems that cannot be solved using conventional linear programming algorithms. Numerical assessments

of probabilities for N events may lead to a linear program with $O(2^N)$ decision variables which, when N is large, is not directly solvable by the simplex method or any other direct method.

Accomplishments

Our research moves on three tracks: development of algorithms and software to support solutions to FTP type problems, investigation of applications of the FTP to realistic problems in reliability and extension of theory to allow incorporation of assertions about qualitative probabilistic structure.

Algorithms

We have constructed an automated method for translating system logic into algebraic inequalities and coupled this method with a column generation algorithm, the Related Integer Program [RIP], which enables us to solve problems of moderate size ($N=150$) with complex logical structure. We prove the existence of an easily computable lower bound on the optimal "Master" LP problem from properties of the RIP, enabling us to control tolerance or deviation from exact optimality easily. This algorithm takes advantage of efficient column generation methods [see Chandra and Hooker (1999, Chapter 4) for a review of column generation methods in this setting.

Jeremy Cohen, a Masters Degree Candidate in Electrical Engineering completed his thesis, *Implementation and Application of the Fundamental Theorem of Probability*, M. Eng. this June. Jeremy's thesis advances the computational capacity of the RIP algorithm and begins investigation of the fundamental problem of incorporating an expert's specification of qualitative probabilistic structure into the de Finetti framework. (See Below).

Fernando Ordonez, a PhD candidate in Operations Research, worked with Professor Rob Freund and me on improvements in algorithms, on theoretical extensions of the FTP and on application of new results to the U.S. Nuclear Regulatory Commission Surry PRA Model.

Theoretical Extensions

Statements about the qualitative structure of uncertain quantities embrace conditional independence and dependence relations, symmetry relations (such as exchangeability) and stochastic order relations. Some of these relations are easily translated into algebraic *linear* equalities and inequalities which then permits application of the FTP in its simplest form. More general types of conditional dependence relations lead to inequalities among multi-linear forms. Logicians working at the interface of mathematical programming and logic have suggested use of non-linear programming methods to deal with the non-linear problem that arises from coupling numerical constraints on probabilities with specification of *qualitative* probabilistic structure. (See Chandra and Hooker *op.cit.* for a discussion.) These methods are cumbersome and computationally difficult.

Bayesian networks, Markov, time-varying and semi-Markov chains are examples of probabilistic systems for which an expert might provide both *numerical appraisal* of some marginal and conditional probabilities as well as information about *qualitative probabilistic dependence relations* governing system events--without fully specifying a joint probability law for all logically realizable events.

Giovanni Andreatta, Professor of Operations Research at the University of Padova and I have designed two algorithms that allow application of efficient column generation techniques designed specifically for LP's *even when the master problem possesses a non-linear objective function and nonlinear constraints*. The ability to do this rests on the following propositions (See *Qualitative Probabilistic Structure and de Finetti's Fundamental Theorem of Probability*)¹.

- Any assertion about the qualitative probabilistic structure of a finite number of dichotomous uncertain quantities can be represented directly as equalities or inequalities among elements of the vector of FTP programming decision variables or *as equalities or inequalities among ratios or among ratios of sums of these decision variables*.
- A conditional dependence relation among N dichotomous uncertain quantities can be represented as a system of *linear* equations in the 2^N or less probabilities of logically possible joint events. This system of

¹ Work on this topic has proceeded past the term of support of July 1996 to September 1998.

equations is indexed by a parameter that assumes values in $[0,1]$ or some subset of it.

These propositions enable us to represent assertions about qualitative probabilistic structure in the form of linear constraints with continuous parameters. We avoid the necessity of dealing with constraints among multi-linear forms and recast the general problem as a *linear* programming problem with a finite number of (continuous) parametric constraints.

These new algorithms perform well on the moderate sized problems tested thus far. They have not yet been tested on large problems ($N > 100$).

Applications

The current version of the RIP algorithm has been tested on several nuclear fault tree examples of moderate size with complex logical structure induced by common mode failure types. [1]

We extend the scope of applications to include Bayesian Networks and Fault Detection Systems that have moderately complex logical structure. Re-analysis of some Bayesian Network problems posed by Pearl and others will allow comparison of alternative methods of analysis. [2]

In particular, we address questions such as, "If experts provide a Bayes Network describing the qualitative structure of a probabilistic system, but do not appraise enough numerical probabilities to allow direct computation of a target event, what does the FTP tell us about upper and lower bounds on the probability of this event given incomplete probabilistic information?" The FTP, coupled with the algorithms cited above, provides a logically sound alternative to backward and forward propagation schemes suggested in the literature.

Other Research

In addition to working in the domain of reasoning with incomplete probabilistic knowledge, I have done research in two other domains.

First, I have established the *exact* distribution of scaled multivariate Normal residuals and the corresponding exact distributions of some related

multivariate statistics. [3] Let x_1, \dots, x_n be a realization of n ($p \times 1$) random vectors (rvs), m be their mean and S be the unscaled sample covariance matrix computed via x_1, \dots, x_n . I prove that the density of a sample standardized version $S^{-1/2}[x_j - m]$ of a generic x_j possesses a simple, spherically symmetric functional form and generalize this result to matrix versions of scaled residuals. This result establishes the exact distribution of a matrix analogue of univariate scaled residuals. The behavior of scaled residuals has long been a benchmark for testing univariate and multivariate normality, so I believe this to be a very useful result. I am circulating the working paper to discover connections in the multivariate literature. Thus far I have found no papers that establish this distribution.

Second, a colleague pointed out that recent research and practice in portfolio analysis goes beyond the traditional (Markowitz—Sharpe) minimization of variance subject to meeting budget constraints and achieving at least a target rate of return. The aim is to distinguish probability of loss from probability of gain. Morgan-Stanley's VALUE AT RISK and the BASLE agreement establishing portfolio management standards for investment banks are examples. In response, I prove that if our aim is to find a portfolio that maximizes a specified fractile of portfolio return subject to achieving *at least* a target rate of return we may be led to *concentrate in place of diversify* in instances where mean-variance analysis yields diversification. [4]

The last piece that I did on software reliability *Successive Sampling and Software Reliability* appeared in the Journal of Statistical Planning and Inference [5].

Personnel Supported

Jun Kent Yan, an MIT MS in Electrical and Ocean Engineering and a second year Sloan Masters degree candidate was supported. Jeremy Cohen's research was done as a requirement for the Masters degree and was not financially supported. Giovanni Andreatta's work was not supported.

Publications

[1] "Reasoning With Incomplete Probabilistic Knowledge Using de Finetti's Fundamental Theorem of Probability: Background and Computational

Issues" with Tracey Myer, and Robert M. Freund. Sloan WP #3990, November 1997 45 pp.

[2] "Exact Distributions of Scaled Normal Residuals", Sloan WP #4000, February 1998 15 pp.

[3] "Qualitative Probabilistic Structure and de Finetti's Fundamental Theorem of Probability" in draft form, 31 pp.

[4] "Concentration not Diversification—Having Our Cake and Eating It, Too!" July 1998 14 pp.

[5] "Successive Sampling and Software Reliability" *J. Stat. Planning and Inference* 49 (1996) 343-369